TD3 : Dérivées

=> Rappel :

Dérivées usuelles :

- où "u" et "v" fonctions quelconques :

$u + v$	$u' + v'$
$u \times v$	$u' \times v + u \times v'$
$\frac{u}{v}$	$\frac{u' \times v - u \times v'}{v^2}$
$1/u$	$\frac{-u'}{u^2}$
$(u)^a$	$a u' \times u^{a-1}$
$\sqrt{u}$	$\frac{u'}{2\sqrt{u}}$
(e^u)	$u' e^u$
$\ln(u)$	$\frac{u'}{u}$

$\boxed{\text{ZGESTION+: 07 66 78 59 45}}$

(i) $f(x) = \ln(2x+4) + e^{2x}$

$\ln(u)' = \frac{u'}{u}$

$u = 2x+4 \quad \Leftrightarrow \quad \frac{u'}{u} = \frac{2}{2x+4}$

$u' = 2$

$(e^u)' = u'e^u$

$u = 2x \quad u' = 2$

$(e^u)' = 2e^{2x}$

$\Rightarrow f'(x) = \frac{2}{2x+4} + 2e^{2x} = \frac{1}{x+2} + 2e^{2x}$

ii) $g(x) = x^{\pi} + 27x - 2$

$\boxed{(x^m)' = mx^{m-1}}$

$(x^{\pi})' = \pi x^{\pi-1}$

$(27x)' = 27$

$\Rightarrow g(x)' = \pi x^{\pi-1} + 27$

iii) $h(x) = \underbrace{e^{x}\left(2x^{4}-x\right)^{1/3}}_{u \times v} - \underbrace{17x^{-5}}_{x^{m}}$

(1) $\quad u \times v \qquad\qquad$ (2) $\quad x^{m}$

1)
$u = e^{x} \qquad\qquad u' = e^{x}$

$v = \left(2x^{4}-x\right)^{1/3} \qquad v' = 1/3\left(8x^{3}-1\right)\left(2x^{4}-x\right)^{-2/3}$

$\hookrightarrow (u)^{a} = a u' \times u^{a-1}$

$u \times v = u' \times v + u \times v'$

$= e^{x}\left(2x^{4}-x\right)^{1/3} + e^{x}\, 1/3\left(8x^{3}-1\right)\left(2x^{4}-x\right)^{-2/3}$

(2)
$\left(17x^{-5}\right)' = -5 \times 17x^{-6} = -85x^{-6}$

$\Rightarrow h'(x) = e^{x}\left(2x^{4}-x\right)^{1/3} + 1/3\, e^{x}\left(8x^{3}-1\right)\left(2x^{4}-x\right)^{-2/3} + 85x^{-6}$

iv) $k(x) = \ln\left(\sqrt{\dfrac{x}{x^2-1}}\right)$

$$\ln\sqrt{\frac{x}{x^2-1}} = \ln\left(\frac{x}{x^2-1}\right)^{1/2} = 1/2\,\ln\left(\frac{x}{x^2-1}\right)$$

$$= 1/2\left(\underbrace{\ln(x)}_{(1)} - \underbrace{\ln(x^2-1)}_{(2)}\right)$$

On se ramène à une fonction simplifiée.

$$(1) : \Big(\underbrace{1/2}_{u}\ \underbrace{\ln(x)}_{v}\Big)' = \underbrace{1/2 \times 1/x}_{\text{que } u \times v' \text{ car } u' = 0} = 1/2x$$

$$(2)\ \big(1/2\,\ln(x^2-1)\big)' = 1/2 \times \frac{2x}{x^2-1} = \frac{2x}{2x^2-2} = \frac{x}{x^2-1}$$

Donc :

$$k'(x) = 1/2x - \frac{x}{x^2-1} = \frac{x^2-1}{2x(x^2-1)} - \frac{2x \times x}{2x(x^2-1)}$$

$$= \frac{x^2-1-2x^2}{2x(x^2-1)} = \boxed{\frac{-x^2-1}{2x(x^2-1)} = k'(x)}$$

(v) $f(x) = \dfrac{x^4 - 2x + 3}{x - 2}$

$$\left(\frac{u}{v}\right)' = \frac{u' \times v - u \times v'}{v^2}$$

$u = x^4 - 2x + 3 \qquad v = x - 2$

$u' = 4x^3 - 2 \qquad v' = 1$

$$\Leftrightarrow \frac{(4x^3 - 2)(x - 2) - (x^4 - 2x + 3) \times 1}{(x-2)^2}$$

$$\Leftrightarrow \frac{4\cancel{x^4} - 8x^3 - \cancel{2x} + 4 - \cancel{x^4} + \cancel{2x} - 3}{(x-2)^2}$$

$$\Leftrightarrow \boxed{\frac{3x^4 - 8x^3 + 1}{(x-2)^2} = f'(x)}$$

(vi) $g(x) = \underbrace{\sqrt{\ln(x+5)}}_{(1)} - \underbrace{4\dfrac{2}{x+4}}_{(2)}$

(1) $\left(\sqrt{\ln(x+5)}\right)' \quad \dfrac{u'}{2\sqrt{u}}$ $= \dfrac{\frac{1}{x+5}}{2\sqrt{\ln(x+5)}} = \dfrac{1}{2(x+5)\sqrt{\ln(x+5)}}$

(2) $-4 \times \dfrac{2}{x+4} = \dfrac{-8}{x+4} =$ $u = -8 \quad v = x+4$

$u' = 0 \quad v' = 1$

$\Leftrightarrow \dfrac{+8 \times 1}{(x+4)^2}$

$$g'(x) = \frac{1}{2(x+5)\sqrt{\ln(x+5)}} + \frac{8}{(x+4)^2}$$

Exercice 2

=> Rappel

Le gradient se définit tel que :

$$\boxed{\vec{\nabla} f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right)}$$

$\frac{\partial f}{\partial x}$: c'est la dérivée de f seulement en fonction de $\boxed{x}$

$\frac{\partial f}{\partial y}$: c'est la dérivée de f seulement en fonction de $\boxed{y}$

$$\boxed{\begin{array}{l}\text{ZGESTION+ :}\\ \text{07 66 78 59 45}\end{array}}$$

(i) $f(x,y) = 2x^2 - xy^3 + 3y^2 + 6xy - 10y^{-2} + 500$

$$\frac{\partial f}{\partial x} = 4x - y^3 + 6y$$

$$\frac{\partial f}{\partial y} = -3xy^2 + 6y + 6x + 10y^{-2}$$

$$\vec{\nabla} f(x,y) = \left(4x - y^3 + 6y \; ; -3xy^2 + 6y + 6x + 10y^{-2}\right)$$

ii) $g(x,y) = \ln \dfrac{x+4y}{2x^2+y}$

$\dfrac{dg}{dx} \Leftrightarrow u = \dfrac{x+4y}{2x^2+y} \qquad u'_{(x)} = \dfrac{1 \times 2x^2+y-(x+4y)(4x)}{(2x^2+y)^2}$

$$\frac{u'_{(x)}}{u_{(x)}} \Leftrightarrow \frac{\dfrac{1 \times 2x^2+y-(x+4y)(4x)}{(2x^2+y)^2}}{\dfrac{x+4y}{2x^2+y}}$$

$$\Leftrightarrow \frac{\dfrac{2x^2+y-4x^2-16xy}{(2x^2+y)^2}}{\dfrac{x+4y}{2x^2+y}}$$

$$\Leftrightarrow \frac{-2x^2+y-16xy}{(2x^2+y)^2} \times \frac{2x^2+y}{x+4y}$$

$$\Leftrightarrow \frac{-4x^4+\cancel{2yx^2}-32x^3y-\cancel{2yx^2}+y^2-16xy^2}{(2x^2+y)^2(x+4y)}$$

$$\frac{dg}{dx} \Leftrightarrow \boxed{\frac{-4x^4-32x^3y+y^2-16xy^2}{(2x^2+y)^2(x+4y)}}$$

$\frac{dg}{dy}$ (∴) $u = \frac{x+4y}{2x^2+y}$ $\qquad u' = \frac{4(2x^2+y)-(x+4y)(1)}{(2x^2+y)^2}$

$$\Leftrightarrow \frac{\dfrac{4(2x^2+y)-(x+4y)}{(2x^2+y)^2}}{\dfrac{x+4y}{2x^2+y}}$$

$$\Leftrightarrow \frac{8y^2+4y-x-4y}{(2x^2+y)^2} \times \frac{2x^2+y}{x+4y}$$

$$\Leftrightarrow \frac{16x^2y^2+\cancel{8x^2y}-2x^3-\cancel{8x^2y}+8y^3+\cancel{4y^2}-xy-\cancel{4y^2}}{(2x^2+y)^2(x+4y)}$$

$$\Leftrightarrow \frac{16x^2y^2-2x^3+8y^3-xy}{(2x^2+y)^2(x+4y)}$$

$$\Rightarrow \vec{\nabla} g(x,y) = \left(\frac{-4x^4 - 32x^3y + y^2 - 16xy^2}{(2x^2+y)^2(x+4y)} ; \frac{16x^2y^2 - 2x^3 + 8y^3 - xy}{(2x^2+y)^2(x+4y)} \right)$$

iii) $h(x;y) = \ln(\underbrace{x+3y^2}_{u}) - ye^{-xy}$

$e^u = u'e^u$

$$\frac{dh}{dx} = \frac{1 \leftarrow u'(x)}{\underbrace{x+3y^2}_{u}} + y^2 e^{-xy}$$

$$\frac{dh}{dy} = \frac{6y \leftarrow u'(y)}{\underbrace{x+3y^2}_{u}} + yx e^{-xy}$$

$$\Rightarrow \vec{\nabla} h(x;y) = \left(\frac{1}{x+3y^2} + y^2 e^{-xy} \; ; \; \frac{6y}{x+3y^2} + yx e^{-xy} \right)$$

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iv) $u(x;y) = \underbrace{e^{3x^2-y}}_{U} \underset{\times}{\times} \underbrace{(4y+7)^5}_{V}$

$\dfrac{\partial u}{\partial x}$: $U = e^{3x^2-y}$ $\quad V = (4y+7)^5$

$U' = 6xe^{3x^2-y}$

$V' = 0$

$\underbrace{U \times V}_{(x)} = 6xe^{3x^2-y} \times (4y+7)^5 + e^{3x^2-y} \times 0$

$= 6xe^{3x^2-y} \times (4y+7)^5$

$\dfrac{\partial u}{\partial y}$: $U = e^{3x^2-y}$ $\quad V = (4y+7)^5$

$U' = -e^{3x^2-y}$ $\quad V' = 5 \times 4 \times (4y+7)^4$

$= 20(4y+7)^4$

$(U \times V)_{(y)} = -e^{3x^2-y} \times (4y+7)^5 + e^{3x^2-y} \times 20(4y+7)^4$

$= -e^{3x^2-y}(4y+7)^5 + 20e^{3x^2-y}(4y+7)^4$

$\vec{\nabla} u(x,y) : \left(6xe^{3x^2-y}(4y+7)^5 ; -e^{3x^2-y}(4y+7)^5 + 20e^{3x^2-y} \times (4y+7)^4\right)$