$$\frac{\partial^2 v}{\partial x^2} = 6xy - \frac{1}{2}(y+2x)^{1/2-1} \times 2$$

$$= 6xy - (y+2x)^{-3/2}$$

$$\frac{\partial v}{\partial x \partial y} = 3x^2 - \frac{1}{2}(y+2x)^{-3/2}$$

$$\frac{\partial^2 v}{\partial y^2} = -\frac{1}{4}(y+2x)^{-3/2}$$

Ex 3)

1) $D_f = \{x \geq 0\}$ → faut que ln soit définie.

2) $f'(x) = u'v + uv'$

$= x^2 \ln(x) + \frac{1}{3} x^3 \frac{1}{x}$

$= x^2 \ln(x) + \frac{1}{3} x^2$

$= x^2 \left(\ln(x) + \frac{1}{3}\right)$

3) $f''(x) = u'v + uv'$. $u = x^2$, $u' = 2x$, $v = \ln(x) + \frac{1}{3}$, $v' = \frac{1}{x}$.

$= 2x\left(\ln(x) + \frac{1}{3}\right) + x^2 \frac{1}{x}$.

$= 2x \ln(x) + \frac{2}{3} x + x$

$= 2x \ln(x) + \frac{5}{3} x.$

4) On etudie signe f'

- $x^2 \geq 0$ tjrs sur $x \geq 0$.
- $\ln x + \frac{1}{3} > 0 \iff \ln(x) > -\frac{1}{3}$

$\iff x > e^{-1/3}$.

x	0		$e^{1/3}$		$+\infty$
x^2		$+$		$+$	
$\ln x + \frac{1}{3}$		$-$	0	$+$	
f'		$-$	0	$+$	
f		↘		↗	

Ex 2)

1) $D_u = \{y + 2x \geq 0\}$

puisqu'il faut que $\sqrt{y+2x}$ soit def.

2) $\frac{\partial u}{\partial x} = 3x^2 y + \frac{1}{2}(y+2x)^{1/2 - 1} \times 2 - 2$

$= 3x^2 y + (y+2x)^{-1/2} - 2$

$\frac{\partial u}{\partial y} = x^3 + \frac{1}{2}(y+2x)^{1/2}$

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Ex 6)

1) $f(x;y) = \sqrt{x-y}$ → nn affine → $\mathcal{D}_f = \{x-y \geq 0\} \to \mathbb{R}^2$.

2) $g(x;y) = x-y+3$ → oui $\frac{\partial g}{\partial x} = 1$ et $\frac{\partial g}{\partial y} = -1$ constante.

3) $u(x;y) = 2e^{-x} - 4x$ → nn $u'(x;y) = 2e^{-x} - 4$

4) $v(x) = \sqrt{x^2} = |x|$ → nn, pas dérivable en $x = 0$

Ex 5)

Colinéaire → $b = \begin{pmatrix} s+1 \\ 2 \\ 1 \end{pmatrix} \neq 0$ puisque 2 de ces coordonnées sont $\neq 0$.

On cherche ensuite $k \in \mathbb{R}_+$ q. $\vec{a} = k\vec{b}$.

$$\begin{cases} 1 = k(s+1) \\ s = 2k \\ -t = k \end{cases} \iff \begin{cases} 1 = k(2k+1) = 2k^2 + k \\ 2k^2 + k - 1 = 0 \\ \Delta = 9 \end{cases} \quad k_1 = \frac{-1-3}{4} = -1, \quad k_2 = \frac{1}{2}$$

$$\begin{cases} s_1 = 2k_1 = -2 \\ t_1 = -k_1 = 1 \end{cases}$$

$$\begin{cases} s_2 = 2k_2 = 1 \\ t_2 = -k_2 = -\frac{1}{2} \end{cases}$$

Orthogonaux → $\langle \vec{a}, \vec{b} \rangle = 1 \times (s+1) + 2s - t$

$= s + 1 + 2s - t$

$= 3s + 1 - t$

Donc $\vec{a}$ et $\vec{b}$ orth si $\boxed{3s + 1 - t = 0}$

Ex 4) $\|\vec{u}\| = \sqrt{1^2 + 3^2 + (-2)^2} = \sqrt{1+9+4} = \sqrt{14}$.

$\mathcal{D} = \{P + t\vec{a} : t \in \mathbb{R}\}$

$$= \left\{ \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} t \in \mathbb{R} \right\} = \left\{ \begin{pmatrix} t \\ 3t+1 \\ -2t \end{pmatrix} t \in \mathbb{R} \right\}$$

Aussi $\begin{cases} x = t \\ y = 3t + 1 \\ z = -2t \end{cases}$ $(t \in \mathbb{R})$.